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# Gain Scheduling Controller for Missile Flight Control Problem by Applying LQR

H. Ghasemzadeh Ebli, A. Khani, and A. Azizi

Department of Electrical Engineering, Tabriz University, Tabriz, Iran
Emails: {*h.ghasemzadeh90, A.khani90, A.azizi90*}*@ms.tabrizu.ac.ir*

***Abstract*** – *In recent years, particularly by increasing the importance of a missile defense technology and its application in various fields, many scientists have been attracted to this position. The actual model of a missile system is nonlinear, and the dynamical behaviors of nonlinear control systems are related to changes in the operating range. In this paper at first we used the Gain scheduling controller to control the flight model and then by applying the linear quadratic regulator (LQR), which is an optimal control method, the system response is studied. Because of high flexibility of quadratic function, use of the LQR has been increased. Here after applying the optimization method we found the under shoot in the result of system missed, thus the system stability is guaranteed.*

***Keywords****: Gain scheduling controller, Missile flight controller problem, LQR control, type-1 servo system*

ORIGINAL ARTICLE

## INTRODUCTION

This is a well-known fact that the applied Engineering Systems are nonlinear and the dynamic behavior of these control systems changes with the operating range. A design method for controlling this behavior is called Gain scheduling. Recently, many studies have been done evaluating the operation of Gain scheduling controller on linear and nonlinear systems and several studies have been drawn to gain scheduling function in flight control problems. According to this, Gain scheduling theory can be successful designing method in engineering branches. Despite the common use of gain scheduling theory, there are some confinements. For example, one or more external variables indicate the existence situation, while scheduled gain depends to the slow values of these variables. The studies in [5] express that gain scheduling is necessarily limited to slow variation in scheduling variables. This limitation is emerged in simulation step and [4], [5] justify this limitation with 5 math formulas.

In this paper, to design an imaginary missile model the gain scheduling technique is used. This is the same missile studies in [2] and [3], but using type-1 servo system instead of $H_\infty$ controller in [2]. After word, carrying out LQR control as optimum controller method, the response of system is verified. Because of high flexibility of quadratic function, use of the LQR has been increased. That discussed in [1].

In continues we discussed the missile flight control model in Section 2 Gain scheduling controller design in Section 3 and also LQR control in Section 4.

## MISSILE FLIGHT CONTROL MODEL

When the missile is flying with an angle of attack ($\alpha$) lift was increased. Perhaps this lift can show acting in the Centers of pressure. This missile close to position of Centers of pressure can be statically stable or unstable. This object studies in [2]. In this paper, the problem that we focus on is that of controlling this missile to track commanded normal acceleration by generating a tail fin defection angle. The missile that we required to design will be accepted a normal acceleration from some external guidance systems and we should know it some of the missile's variables measured by gyros and accelerometers. A Missile model and the missile-air frame control model shown in Fig. 1 and Fig. 2, respectively.

Fig. 1 - A missile model

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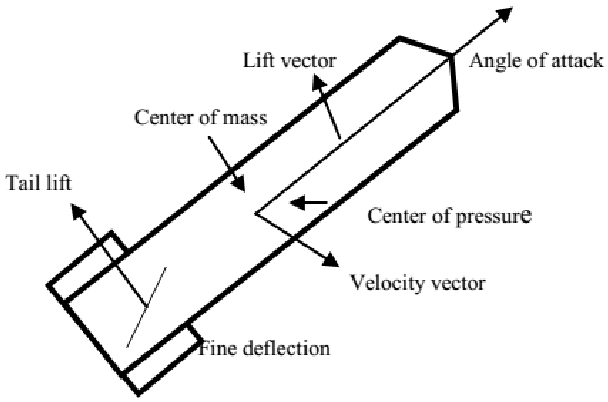


Fig. 2 - Missile flight control model

### A. *Mathematical description of Missile model*

A missile flight control model using in this paper has been illustrated in Fig. 3.

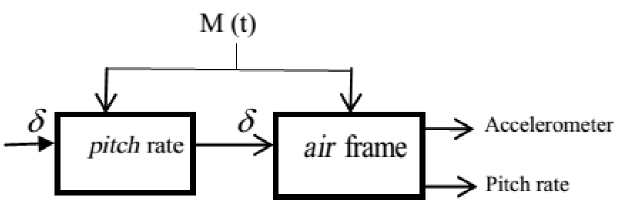


Fig. 3 - The block diagram of missile model.

**Air frame dynamics**

$$\dot{\alpha}(\mathrm{t}) = \mathrm{K}_{\alpha} M(\mathrm{t})\,\mathrm{C}_{n}[\alpha(\mathrm{t}), \delta(\mathrm{t}), \mathrm{M(t)}]\cos(\alpha(\mathrm{t}) + \mathrm{q(t)})$$

$$\dot{q}(\mathrm{t}) = \mathrm{K}_{q} M^{2}(\mathrm{t})\,\mathrm{C}_{m}[\alpha(\mathrm{t}), \delta(\mathrm{t}), \mathrm{M(t)}]$$

**Actuator Dynamics**

$$\frac{d}{dt}\begin{bmatrix} \delta(\mathrm{t}) \\ \dot{\delta}(\mathrm{t}) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_a{}^2 & -2\zeta\omega_a \end{bmatrix}\begin{bmatrix} \delta(\mathrm{t}) \\ \dot{\delta}(\mathrm{t}) \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_a{}^2 \end{bmatrix}\delta_c(\mathrm{t})$$

**Out put**

$$\eta_z(t) = K_z M^{2}(t) C_n[\alpha(t), \delta(t), M(t)]$$

**Variables**

$\alpha(\mathrm{t})$ =angle of attack, range $-20 \le \alpha \le 20$ .

$M(\mathrm{t})$ =Mach number, range $2 \le M \le 4$ **.**

$q(\mathrm{t})$ =pitch rate.

$\delta_c(\mathrm{t})$ =command tail fin deflection angle.

$\delta(\mathrm{t})$ =actual tail fin deflection.

$\eta_c(\mathrm{t})$ =command normal acceleration.

$\eta_z(\mathrm{t})$ =actual normal acceleration.

**Simulation Variable**

For simulation purposes, a state equation for the Mach number is defined as follows:

$$\dot{M}(\mathrm{t}) = \frac{|\eta_z(\mathrm{t})|}{|\cos(|\alpha(\mathrm{t})|)|}\sin(|\alpha(\mathrm{t})|) + \mathrm{A}_x M^2 \cos(|\alpha(\mathrm{t})|)$$

**Aerodynamic Coefficients**

$C_n[\alpha(t), \delta(t), M(t)]=$

$$\mathrm{sgn}(\alpha(t))\left[a_n|\alpha(t)|^3 + b_n|\alpha(t)|^2 + c_n(2 - \tfrac{1}{3}M(t))|\alpha(t)|\right] + d_n\delta(t)$$

$C_m[\alpha(t), \delta(t), M(t)]=$

$$\mathrm{sgn}(\alpha(t))\left[a_m|\alpha(t)|^3 + b_m|\alpha(t)|^2 + c_m(-7 + \tfrac{8}{3}M(t))|\alpha(t)|\right] + d_m\delta(t)$$

## GAIN SCHEDULING CONTROLLER DESIGN

In this section, we are designing missile flight control system with gain scheduling techniques and also we will be discussed about it. By using gain scheduling controller, missile design process is divided into the 4 following step:

1. Choosing the equilibrium.
2. Linearization system around each equilibrium point.
3. Linear controller design.
4. Scheduling the set of linear controllers.

### *A. Choosing the equilibrium*

To determine the equilibrium points of this system according to definition, we putted $f(\mathrm{x,u,y}) = 0$ . Thus, set of the operating points are shown as follows:

$$\delta(\alpha, \mathrm{M}) = -\frac{1}{d_m}\mathrm{sgn}(\alpha)[\mathrm{a}_m|\alpha|^3 + \mathrm{b}_m|\alpha|^2 + \mathrm{c}_m(\frac{8}{3}\mathrm{M(t)} - 7)|\alpha|]$$

$$q(\alpha, M) = -K_a M C_n[\alpha, \delta(\alpha, M), M]\cos(\alpha)$$

$$\dot{\delta}(\mathrm{t}) = 0$$

$$\delta_c(\alpha, \mathrm{M}) = \delta(\alpha, \mathrm{M})$$

### *B. Linearization system around each equilibrium point*

To linearize the nonlinear system, we use the Taylor series expansion of **f** and **g** around an equilibrium state and withdraw ling the high-order term of order greater than2. Then by using the Jacobin matrices values of $\mathrm{A}(\alpha, \mathrm{M}), \mathrm{B}(\alpha, \mathrm{M})$ and $C(\alpha, \mathrm{M})$ are calculated as

$$A(\alpha, \mathrm{M}) = \begin{bmatrix} K_\alpha M[\mathrm{Cn}_\alpha\cos(\alpha) - (\mathrm{C}_n + \mathrm{d}_n\,\delta(\alpha, \mathrm{M}))\sin(\alpha)] & 1 & \mathrm{K}_\alpha M d_n\cos(\alpha) & 0 \\ \mathrm{K}_q M^2 Cm_\alpha & 0 & \mathrm{K}_q M^2 d_m & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\omega_a^2 & -2\zeta\omega_a \end{bmatrix}$$

$$B(\alpha, \mathrm{M}) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \omega_a^2 \end{bmatrix}$$

$$C(\alpha, \mathrm{M}) = \begin{bmatrix} K_z M^2 Cn_\alpha & 0 & \mathrm{K}_z M^2 d_n & 0 \end{bmatrix}$$

*C. Linear controller design*

The design of the controller, for controlling a nonlinear system, it is necessary that, the controller design process is divided into two steps: At first for linearization nonlinear system, we should design several local controllers around each equilibrium points. In second step, we should schedule and interpolated gain of the local designated. In continues, linear controller designated process was explained. To design this controller, the nonlinear system used in part 3.2 is considered. Thus, it should be designed around several operating points for linearized systems. Here we have three operating points $(10^\circ,2),(10^\circ,3)$ and $(10^\circ,4)$ for $(\alpha,\mathrm{M})$.The system is mostly on penetration of the match number as in [2]. Thus, choosing an attack angle $10^\circ$ for each three match numbers seems to be a wise compatibility and this angle states the average point of operating range. The first aim of this step is to fix the match number that stabilized the controller in all attack angles between $-20^\circ < \alpha < 20^\circ$. While the symmetric property of system makes it possible to verify the system under attack angles between $0^\circ < \alpha < 20^\circ$. Type-1 servo system that is on the basis of pole placement is used to design a desirable controller. As some state variables are not available, observer state is placed in this kind of controller systems. This is described as follows:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\tilde{x}}(t) \\ \dot{\xi}(t) \end{bmatrix} = \begin{bmatrix} A & \text{-BK} & \mathrm{BK}_i \\ LC & \text{A-BK-LC} & \mathrm{BK}_i \\ -C & 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \tilde{x}(t) \\ \xi(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(t)$$

$$y(t) = [C \quad 0 \quad 0] \begin{bmatrix} x(t) \\ \tilde{x}(t) \\ \xi(t) \end{bmatrix}$$

Here, gain K from the pole placement design and gain L from the state observer have been calculated. We have the step response of liner closed loop system for three constant operating points, in Fig. 4. Which is indicates that, output of the system follows the step response with constant time less than 25 S.

In the following, in Fig. 5 we presented the frequency response of the open loop system in the same operation range that shown for each $300\,rad/\sec$ frequency all amounts should be lower than -30 dB, considering the following figures.

*D. Scheduling the set of linear controllers*

Essentially, gain scheduling technique is divided into two stages: First, design a local controller based on linearization nonlinear system, around the each equilibrium points, which are described in the previous Section.

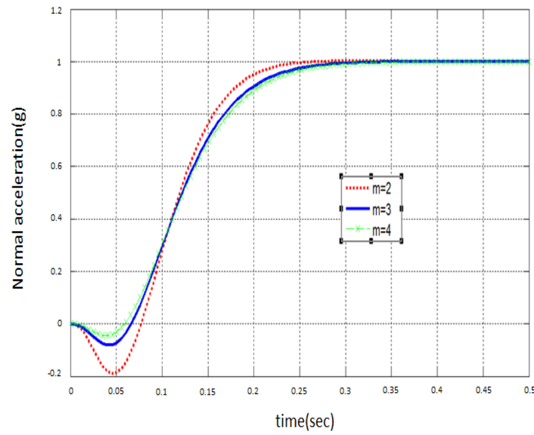


Fig. 4 - Step response of the closed-loop system for $\alpha = 10^\circ$

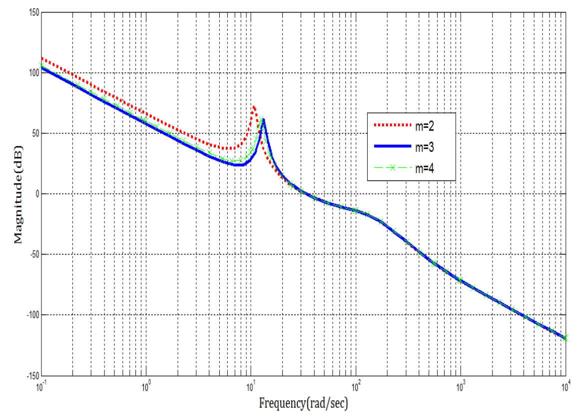


Fig. 5 - Frequency response of the open loop system for $\alpha = 10^\circ$

Local controllers were designed in three operation point's $((\alpha,\mathrm{M}) = ((10^\circ,2),(10^\circ,3),(10^\circ,4))$.Each equilibrium point, gives a special gain that makes the controller capable of satisfying the system requirements locally around each design point. The second step, to be discussed in this chapter, requires interpolating, or scheduling, the gain of the linear designs to obtain a nonlinear controller. The three specified controller are rewritten as follow tables:

**Table 1**

**The gain from the local controller at fixed Mach number M=2**

| **At fix Mach number M=2** |
|---|
| KK2=[-5.7137e+00,-4.5613e-01,-4.0079e-01,-1.1296e-03] |
| Ki2=6.3689e+01 |
| KL2=[-7.7941e+02,-7.6420e+04,2.0806e+03,-2.5828e+05] |

**Table 2**
**The gain from the local controller at fixed Mach number M=3**

| At fix Mach number M=3 |
|---|
| KK3=[-2.2529e+00,-1.7227e-01,-4.0539e-01,-1.1405e-03] |
| Ki3=1.4099e+01 |
| KL3=[-4.0547e+02,-3.9609e+04,9.1918e+02,1.16157e+05] |

**Table 3**
**The gain from the local controller at fixed Mach number M=4**

| At fix Mach number M=4 |
|---|
| KK4=[-1.3009e+00,-9.5897e-02,-4.0555e-01,-1.1457e-03] |
| Ki4=5.0734e+00 |
| KL4=[-2.7506e+02,-2.6813e+04,5.1287e-02,-6.6569e+04] |

The results of linear controller analysis shows that, gains in different constant Mach number can create stable linear system on fixed value and in all range of attack angle between 0 to 20. Regarding this problem the challenge to schedule all gains in different match number values was considers this means, the gain can be define is in range $2 \le M \le 4$.

## LINEAR QUADRATIC REGULATOR (LQR)

Linear quadratic regulator (LQR) is an optimal control which is an optimal control method, the system response is studied. Because of high flexibility of quadratic function, use of the LQR has been increased.

Quadratic function is defined in LQR as follows:

$$J_c = \frac{1}{2} x^T(t_f) G x(t_f) + \frac{1}{2}\int_{t_0}^{t_f} (x^T Q_c x + u^T R_c u)\,dt$$

As the optimization should be consider completely in all time ranges and not dedicated to any special time, set $t_f = \infty$ and $t_0$ equals zero. Here, ultimate values of states aren't very significant and goal is to confine the states in low values in all times. Thus, the **G** matrix should be zero. According to this, we have the quadratic equation like follow:

$$J_c = \frac{1}{2}\int_0^{\infty} (x^T Q_c x + u^T R_c u)\,dt$$

For system $\dot{x} = Ax + Bu$ the state feedback controller minimizing quadratic function is:

$$u = -Kx$$

which

$$K = R_c^{-1} B^T P_c$$

And $P_c$ originating from Recaty function in stable state is:

$$A^T P_c + P_c A - P_c B R_c^{-1} B^T P_c + Q_c = 0$$

Assuming that:

$$\forall i : x_{i\max} = 0.005 \qquad \& \ \forall j : u_{j\max} = 0.05$$

Weight matrixes equal

$$Q_c = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 100 \end{bmatrix}$$

$$R_c = 1$$

$$K_{lqr} = [88.1308, -2.6403, 54.0717, 9.9909]$$

As a result, the optimization controller can set the poles of closed loop system into the minimum values of the quadratic function.

## SIMULATION

The way of our research in this paper is that, we designed missile flying controller model with gain scheduling controller. After that by adding the LQR we found the optimization response for this nonlinear system. We consider the following constant values to reach the purpose of simulation. For constant Match number 2, 3 and 4 we simulate the system using gain scheduling controller as shown in Fig. 6, and once again by applying linear quadratic function (LQR) as shown in Fig. 7.

$$K_\alpha = (0.7) P_0 S / m v_s$$
$$K_q = (0.7) P_0 S_d / I_y$$
$$K_z = (0.7) P_0 S / m$$
$$A_x = (0.7) P_0 S C_a / m$$

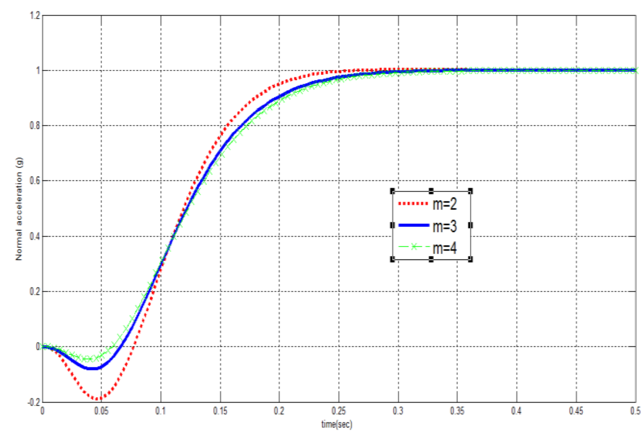


Fig. 6 - Step response of closed loop system with pole placement controller

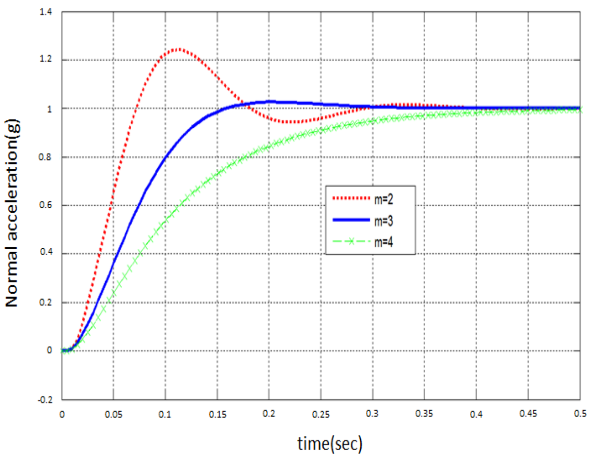


Fig. 7 - Step response of closed loop system by applying optimization control LQR

The results of the comparison from the above figure are that, the optimized system guarantees stability have been increased and reason of that conversion the non-minimal state response to minimal one.

## RESULTS AND DISCUSSION

In this paper, missile designing was discussed using gain scheduling and then LQR method was added to system. As mentioned before said the gain scheduling is fictional in engineering branches, it has some limitation is exogenous variables that vary very suddenly.

Then the Match number was considered as an exogenous variable and the all math definition of the missile model was submitted beside the Match number state equation (The match number equation is not a proper missile system but has a very fundamental role is simulation).

Then the linear controller designated at first by linear systems in three distinct constant operation points and also by LTI technique. These controllers can guarantee the local performance and nominal stability of system.

The frequency response of the open-loop linear system indicated that the system value is lower than 30 dB in 300 rad/sec.

Finally linear quadratic regulator was defined and after applying above optimization method we found the under shoot in the result of system missed, thus the system stability is guaranteed.

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