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Graph Coloring Problem based on Discrete Social Based Algorithm

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Abstract *– A new and popular technique for combinatorial optimization is to embed local search into the framework of evolutionary algorithms. In this article we propose a discrete hybrid evolutionary algorithm of the traditional genetic algorithm followed by the new Imperialist Competitive Algorithm (Social Based Algorithm) to solve the graph-coloring problem. A good selection for graph coloring is a function of vertices of graph into a set of colors such that both neighbor vertices of the graph have different colors. Using the Social Based Algorithm makes a new technique with a high accuracy in both exploration and exploitation in the search space for coloring the graph. The algorithm is tested toward different standard benchmark tests while limiting the number of usable colors to the known chromatic numbers. Results prove the superiority of the proposed method in comparison with other algorithms and even in terms of the minimum number of colors required for graph coloring problems.*

Keywords*: Graph coloring problem, Social Based Algorithm, Evolutionary algorithms.*

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INTRODUCTION

The Graph Coloring Problem (GCP) is a popular NP-complete problem. This problem contains both edge coloring and vertex coloring. The target in the vertex coloring is to color a number of vertexes so that if two vertices are connected in the graph, they will be colored with different colors. Therewith, the number of different colors that can be used to color the vertices is restricted and a secondary objective is to find the minimum number of various colors required to color a specific graph without violating the neighbor constraint. This minimum number is known as the Chromatic Number ($\chi(G)$) [1].

If k = {1, 2, 3...} and P (G, k) is the number of possible solutions for coloring the graph G with k colors, then

$$\chi(G) = \min_{k}(P(G,k) > 0) \tag{1}$$

Basically, coloring problems of graphs are very popular from the theoretical standpoint since they are a class of NP graph coloring problems are very interesting from the theoretical standpoint since they are a class of NP complete problems that also belong to Constraint Satisfaction Problems (CSPs). Graph Coloring has several applications which are not limited to:

- Map coloring [2]
- Scheduling [3]
- Radio Frequency Assignment [4, 5]
- Register allocation [6]
- Pattern Matching [7]
- Sudoku [8]

In this paper we proposed the use of social based algorithm to solve the graph-coloring problem with assuming that the usage of no more than the number of colors equal to the chromatic index to color the test graph. The chapter is organized as follows: In Section 2, a brief review of related prior works to solve the graph coloring problem using optimization methods are presented. In Section 3, the standard SBA model is introduced and the modification for using in coloring the graph is explained exactly. The fitness function for introducing the coloring graph as an optimization problem is then presented in Section 4. The performance of the newly developed algorithm is then compared by the performance of conventional genetic and imperialist competitive algorithms in Section 6 before Section 7 concludes with final remarks.

RELATED WORKS

There are a great number of research has been done to analyze the theoretical aspects of the Graph Coloring Problem in terms of its generalization as a Constraint Satisfaction Problem [9].

Using evolutionary algorithms to solve the graph-coloring problem has been detailed in a number of papers including ones by Back, Hammel and Schwefel [10] as well as work by Eiben et al. [11].

A great deal applications in the real-world, the arrangement, ordering, and selection of a discrete set of objects from a finite set, is used to satisfy a desired purpose. The problem of finding optimal syntax from a discrete set of objects is known as the combinatorial optimization problem [12].

Provided many of the combinatorial problems can be optimized in polynomial time, a large number belong to the class of NP -hard [13]. Heuristic algorithms have been utilized to deal with these hard combinatorial optimization problems as collusion between solution quality and computational time [14].

Meta-heuristic algorithms as recent developed classes of heuristic algorithms have performed promising results in the field of combinatorial optimization. The class of meta-heuristic algorithms comprises (but not restricted to) simulated annealing (SA), ant colony optimization (ACO), tabu search, genetic algorithms (GAs), bacterial foraging [15], etc.

Recently, a new approach of computationally efficient meta-heuristic algorithms has been developed. This approach combines the Evolutionary Algorithm (EA) and socio-political process based Imperialist Competitive Algorithm (ICA) to enhance handling non-linear constraints and non-convex solution spaces [16].

This approach tries to capture several people involved in community development characteristic. People live in different type of communities: Monarchy, Republic, Autocracy and Multinational. Leadership styles are different in each community. SBA has been undertaken to deal with curse of dimensionality and to enhance the convergence speed and accuracy of the basic ICA and EA algorithms [16].

While SBA has proven to be successful on a variety of continuous functions, restricted success has been illustrated to adapt SBA to more complex richer spaces such as combinatorial optimization.

In this article, the concepts of the standard SBA model are expanded to the discrete combinatorial space and a new SBA is developed to solve the combinatorial optimization problem.

SOCIAL BASED ALGORITHM

The Social-Based Algorithm (SBA) is improved by Ramezani and Lotfi in 2013 [16]. SBA algorithm is a new combinational algorithm which makes utilization of mechanisms inspired by incorporating both Evolutionary Algorithm (EA) [19] and socio-political process based Imperialist Competitive Algorithm (ICA) [18]. SBA algorithm because of using both biological and social evolutions has a high performance in searching the search space, preventing in fast converging and escaping from the trap of local minima [16].

*A. **Evolutionary Algorithm(EA)***

An Evolutionary Algorithms (EA) is a generic population-based meta-heuristic optimization algorithm which employs biology-inspired mechanisms like mutation, crossover, natural selection, and survival of the fittest in order to develop a set of solution candidates iteratively [17].

EAs share a common idea. The idea of EA is based on survival of fittest and it makes a rise in the fitness of the population in different generations. Based on the fitness function some of the better candidates are selected, they seed the next generation by implementing recombination and mutation. Performance of these operators leads to a set of new candidates, the offspring. Replacement operator changes new offspring in next generation, based on the fitness.

B. ***Imperialist Competitive Algorithm***

Imperialist competitive algorithm (ICA) commences by producing a set of candidate random solutions in the search space of the optimization problem. The generated random off springs comprises the initial countries in the world (initial population). Countries are divided into two groups: imperialists and colonies [16]. The more powerful imperialist means the greater number of colonies.

The fitness function of the optimization problem characterizes the power of each country. Among these countries, some of the best initial countries based on their power become Imperialists and start taking control of other countries (called colonies) and form the initial Empires [18].

There are three principal operators for this algorithm: Assimilation, Revolution and Competition. This algorithm utilizes the assimilation policy. The imperialists in this policy attempt to enhance the economy, culture and political situations of their colonies. This policy makes the colony's avidity toward the imperialists. Assimilation operator brings the colonies of each empire get closer to the imperialist state in the space of socio-political characteristics (optimization search space) [16]. Revolution takes about sudden random replaces in the position of some of the countries in the search space. Within assimilation and revolution, a colony might reach a superior position and has the chance to take the control of the all empire and replace the current imperialist state of the empire.

In competition operator, imperialists try to achieve more colonies and the colonies start to move toward their imperialists. All the empires try to win and take possession of colonies of other empires. The power of the empires depends on the power of their imperialist and their colonies. All the empires in iterations have a chance to take control of one or more of the colonies of the weakest empire based on their power. In this competition, the strong imperialists will be improved and the weak ones will be collapsed. After some iteration, the weaker empires will lose their entire colonies and their imperialists will move into the other empires; after all, the weak empires will be collapsed and only one strong empire will be remained [16].

C. ***Social-Based Algorithm***

Like other optimization algorithms, SBA starts by generating a set of random solutions in the search space. The produced candidates are called the initial population and comprise persons. In cultural techniques, people clusters make society and develop the society. A person is an array of size n in an n dimensional optimization problem as below:

$$Person = [x_1, \ldots, x_n] \quad (2)$$

The cost of persons can be achieved by considering the cost function f at the variables $[x_1, \ldots, x_n]$:

$$C_{pi} = f(Person_i) \quad (3)$$

Where C_{pi} is the cost of ith person in the algorithm.

People provide different types of communities: Republic, Monarchy, and Multinational and Autocracy. People territory differs with different society forms [16].

Republic shows a president-based form of the arrays. In this society, each people are chosen for a set period of time. The people with best costs are selected as candidates and the others can vote to their president. If that happens, entire number of votes characterizes the president. Autocracy defines a free-based form of society with no leader. In this society, each people are free and can do whatever it wants.

Monarchy includes a leader-based form of society. In this society, monarch has full authority on its people and people should follow it. Power is divided along through the family. The highest cost of people is chosen as the monarch in each country. Among these different monarchy countries, the best monarch is selected as empire.

Multinational society makes an international production or delivers services [20]. The formations of these communities affirm that the total power belongs to the working group. They generate the products in different areas of the country and send them to the others.

Like other population-based algorithms, SBA starts with a number of random populations which are initialized at the same element position of the conventional array. These populations are called people and have the entire number of N-Person people. The numbers of N-Country of the best people are chosen as leaders and the rest are considered as the people of these countries. These people are achieved after the initial cost fittings. To form the initial countries, the people are divided among the countries based on their strength. Total number of countries can be achieved as below:

$$N_{Country} = N_{Monarchy} + N_{Republic} + N_{Autocracy} + N_{Multinational} \quad (4)$$

The total strength of a country depends on both the leader's strength and the strength of its people. This fact can be modeled by considering the entire strength of a country as the power of the leader of the country plus a percentage of mean strength of its people [16]. The power of each people effects on the total strength of that country:

$$T.P_{ci} = \cos t(leared_i) + \xi \times mean\{\cos t(\text{People of country}_i)\} \quad (5)$$

where $T.P_{ci}$ is the total total strength of ith country and ξ is a positive constant less than 1. Small value for ξ makes the total strength of the leader to be considered by just the leader and incrementing it, will add to the role of the people in characterizing the total power of a leader. After initializing, Evolutionary operators include: crossover, selection and mutation applied on the people of each country. The leader doesn't effect from the introduced operators. After evolutionary operators implemented, ICA operators begin the process with two different operations as below:

a) External operations: these operations are implemented among the countries. Assimilation is utilized just among monarchy countries of each imperialist. When a country moves into an imperialist, it moves its total people into that imperialist. Assimilation implements on just empires, for this reason, it can just apply on the monarchy countries. The movement of the leaders into the empires is done by x units:

$$x \sim U(0, Coeff_{\text{external assimilation}} \times d) \quad (6)$$

$$\theta \sim U(-\gamma, \gamma) \quad (7)$$

where x is a random value restricted in [0, 1] and $Coeff_{\text{external assimilation}}$ is a number greater than 1 and d is the distance between leader and empire. In ICA, the angle of countries is employed to improve the escaping capability from local optima trap by moving imperialist's position. θ is deflection angle and defines the direction of movement, γ represents the deviation from the original direction (in this work $\gamma = \pi/4$ (rad)). Revolution arises in all countries. Total number of the people in a country should move into the same way because revolution is vis-a-vis by the total in monarchy countries. In other countries they try to improve their selves therefore they move with each other.

b) Internal: internal types are among the people of the countries. Assimilation operation occurs in total countries they move into the leaders and revolution operation occurs in all countries, people try to get the position. The external revolution is utilized to one other country with revolution probability Pe and the internal revolution is utilized to one other person with revolution probability Pi. Internal assimilation in multinational communities is outdone in two steps. At first, the person of ith country moves into ith position unto the ith position of its leader (Eq. 8) and the next step is that each part of the point to be produced is spread out over different countries.

$$x_i \sim U(0, \beta \times d_i) \quad (8)$$

where β is a constant ($\beta > 1$) and d_i is the distance between ith position of person and leader and describes Nc countries for societies, because countries optimize their own part of variable. The number of these societies can be evaluated as:

$$N_{Multinational} = k \times N_C \quad (9)$$

where k is a constant. Each country tries to increment its ownership. After generating different parts in multinational society they export their product to the others. In other words, each person take set of [1, . ., i − 1, i + 1, . . . , Nc] parts from the other countries:

$$X_j = leader_j \times x_j \qquad (10)$$

X_j is the jth position of the leader of jth country. This progress describes a simple model of assimilation policy that was followed by some of the imperialist states. Revolution movement gives almost sudden random replacements in the position of the countries in the search space (exploration) and save the problem not to get into local minimum trap and this is one of the reasons why we employed this algorithm. This process also implemented to select better leader as a new empire. In competition, a weak country of the weakest empires makes a competition among all empires to accede this country. In this condition, the competition is achieved by evaluating the cost value of countries. After calculating the cost of the all countries, the country or empire which loses all of its people or countries get be collapsed.

MODIFIED SOCIAL-BASED ALGORITHM

This section describes a discrete version of social-based algorithm which is called DSBA. The basic version of SBA is proposed to solve continuous problems. So with some modifications in some operators of the SBA, it can be employed to solve discrete problems.

In the SBA, the assimilation operator makes the colonies to move into their relevant imperialists. The result of this event is to the colonies become much like to their relevant imperialist states. Imperialists begin to improve their colonies, on the other hand following assimilation policy, the imperialists attempted to attract their colonies and make them a part of themselves. This operator must be altered to utilize in discrete problems. To model the assimilation policy in the discrete social-based algorithm, we used 2-point crossover in assimilation policy. By using crossover, some random parts of imperialists and their relevant colonies are replaced between them. In 2-point crossover operator, both countries (imperialist and a colony) are cut at two arbitrary places and the selected parts of both countries are exchanged among themselves to generate two new countries. Modification of the SBA parameters is described in below:

A. *Initial population*

The initial population is very significant for determining the final solution. The initial population is generated through a random perturbation that specifies |V| random colors to the persons of the countries. Thus, this would result with a definite size of initial population where every vertex is assigned a unique color. Assume that X_{norm} is a number in the interval [0, 1]. To map the presented space in the range [minVal, maxVal], the equation below can be utilized:

$$x = (\max Val - \min Val) \times X_{norm} + \min Val \qquad (11)$$

From the above, we can considered that it is possible to have a decimal number after generating and mapping the produced persons; so, to convert the initial persons to a whole number, floor function can be used where floor makes the produced number to round the decimal number toward negative infinity. We also used a minimum function (min) to guarantee the generating the maximum number. The final equation can be written as below:

$$x = floor(\min(\max Val - \min Val) \times X_{norm} + \min Val)) \qquad (12)$$

With this policy, we can use SBA for the discrete based problems, especially graph coloring. The presented policy is used to generate initial population which includes genes of chromosomes, persons, colonies and imperialists to make the initial values to be a whole integer number.

B. *SBA Operators*

Social based algorithm operators are used to explore the solution space. The operators are employed iteratively in every decade with their corresponding probabilities.

In the proposed algorithm after the GA operators are applied, the final results before entering to the ICA stage, are rounded by the eq.12.

After applying GA parameters, solutions enter to the ICA part. Assume a graph G=(V,E) which should be colored with minimum number of colors; in this graph |V| and |E| illustrate the vertex and edges respectively and are equal to5. This graph is shown in the Figure below. A simple graph coloring is shown below; as it can be seen, the left hand illustrates a non-colored graph with 10 vertex and 5 edges and the right hand illustrates that graph after a simple coloring.

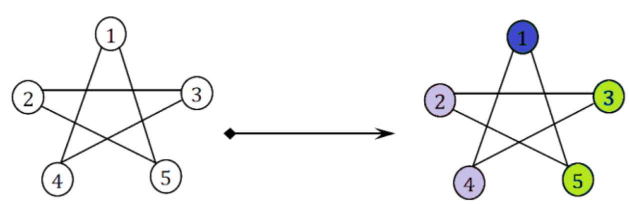

Fig. 1 - Simple graph coloring process; left hand: Graph G before coloring, right hand Graph G after coloring.

By considering the following imperialist and colony countries, the cut points selected randomly and are c1=3 and c2=5. The new generated country is depicted below:

$$\left.\begin{array}{r} imperialist_i : \langle 1,2,\underset{\downarrow}{3},5,\underset{\downarrow}{4} \rangle \\ colony_i : \langle 5,8,\overset{\uparrow}{9},5,\overset{\uparrow}{8} \rangle \end{array}\right\} \Rightarrow new\_colony_i = \langle 5,8,3,5,4 \rangle$$

In the DSBA, in addition the assimilation, the revolution operator also requires to be changed. Revolution operator makes the suddenly variation of the position in the solution space in a country. The revolution operator enhances the exploration power of the SBA and helps it to escape from entrapping in the local optima. The modified revolution operator makes two different cells of a country are chosen and then the selected cells are swapped among themselves. The revolution operator can be demonstrated in the example below:

$$colony_i : \langle 1,\vec{2},3,\vec{4},4 \rangle \Rightarrow new\_colony_i = \langle 1,3,4,2,4 \rangle$$

C. *Cost Function*

In the graph coloring problem, there are two optimization criteria:

1) There is only one color to be devoted to each vertex.

2) Two adjacent vertices are not permitted to have the same color.

Minimize the number of conflicts in the vertex coloring makes to minimize the number of colors. Thus, every time we have a conflict in a person, the person is kept in the solution but a penalty is assigned.

An operative person has a penalty of 0. To achieve an efficient allocation for the penalty in the graph, the persons that have been altered are scanned due to characterize the least number of colors, k, that have been found by the algorithm in proper persons. After that, the algorithm devotes a penalty n to every edge violation (two nodes with the same color). The final cost function is the sum of all penalties:

$$Fitness = \sum_{i=0}^{N} \left(K + \sum_{i=0, i \neq j}^{N} Penalty(i,j) \right) \tag{13}$$

where Penalty(i, j) is equal to n if i and j are connected and 0 otherwise.

Every feasible person in the population refers to a possible solution to the graph coloring problem. However, the population has in addition to the feasible persons, unfeasible ones. During every generation, SBA operators are applied to the solutions.

At first, every node in the graph is randomly allocated a distinct color resulting with |V | coloring. As the algorithm evolves and since the algorithm does not know the chromatic number of the graph, X(G), we squeeze or reduce the number of colors piecemeal every time a feasible coloring with k colors is obtained. The algorithm stops either when the maximum time is exceeded or if the algorithm fails to develop the number of colors after some iteration. The pseudo-code for social based coloring algorithm is illustrated in the below:

Step 1: Initializing parameters;

Step 2:

2.1. Define the GraphColoring;

2.2. Generate some random color index (people);

2.3. Select some powerful random color index as leaders;

2.4. Randomly allocate remain color index to different countries;

2.5. Initialize the empires with imperialists cost function $T.P_{C_i}$.

2.6. Select more powerful leaders as the empires;

Step 3: Decade loop: $N_d = N_d + 1$

Step 4: For $i = 1, 2, \cdots, N_{country}$ do:

4. 1. Modified Crossover;

4. 2. Modified Mutation;

Step 5: For $i = 1, 2, \cdots, N_{imp}$ do

5. 1. Assimilation policy (by using crossover operator): Move the color index as people of each country toward their relevant leaders, using:

d is the distance between person and leader.

5. 2. Modified Revolutionary;

5. 3. Migration; Pick the weakest person (solutions with repetitive colors, etc.) from the weakest country and give it to the rightful country.

5. 4. Countries assimilation policy: Move the leaders of each country toward their empires and move the people of each country as the same as their leaders, using:

d is the distance between leader and imperialist.

5. 5. Countries revolutionary;

5. 6. Competition; Pick the weakest country from the weakest empire and give it to the rightful empire.

5. 7. Elimination; Eliminate the powerless empires.

Step 6: Terminating Criterion Control; Repeat Steps 3-6 until a terminating criterion is satisfied.

RESULTS

The proposed algorithm was implemented using the MATLAB language and tested on various benchmarks on a Intel (R) Core(TM) i5-3230M 2.6 Ghz workstation. The algorithm was tested on the following test instances:

The values of parameters in GA and ICA algorithms for the graph coloring problems are presented in Table 1. These values are selected based on some preliminary trials. Note that the parameters of SBA are equal to these parameters; because it is a combination of these two algorithms.

TABLE 1
PARAMETER SETTINGS [16, 29]

Algorithm	**GA**				**ICA**	
Parameter	Mutation Rate	Crossover Rate	Step Size	Chemotactic Step	α	β
Value	0.9	0.1	1E-7	1000	0.5	2

Dataset for the test instances are graphs from Donald Knuth's Stanford Graph Base and the center for Discrete Mathematics and Theoretical Computer Science via ftp [21].

We tried in this category book graphs where each node considers a character. Two nodes are connected if the corresponding characters encounter each other. We select some random number of three graph category generated based on Anna karenina, David Copperfield, Huckleberry Finn and some Leighton graphs which are random graphs with a fixed number of edges and predefined chromatic number. The other set of graphs in the category are the queen graphs. Given an n × n chessboard, a queen graph is a graph on n2 nodes, each related to a square on the board. Two nodes are connected by an edge if the corresponding squares are in the same row, column, or diagonal.

Table 2 shows the experimental results for the explained graphs. These graphs form a good reference for comparison. Moreover, these graphs are hard to color and form a real challenge for graph coloring algorithms. The first column illustrates the graph benchmark file while the next three columns illustrate the number of graph's vertex and number of edges, In order to solve a given k−coloring instance, algorithms are run for 20 − 30 times with each run.

Convergence diagram of the Algorithm for Myciel6 graph benchmark is shown in below; from the figure, it is seen that the algorithm reach to the minimal point, i.e. minimum number of colors in iteration 5.

Algorithm movement in the case of the Myciel6 graph benchmark for the target is shown below. From the figure, we can see the variations of the graph for reaching the final target.

TABLE 2
ATTEMPTED GRAPH COLORING RESULTS: LEIGHTON GRAPHS, MYCIELSKI GRAPHS, FLAT GRAPHS, QUEEN NXN GRAPHS, AND COURSE SCHEDULING GRAPHS.

Graph Benchmark	No. of Vertex	No. of Edges	GA		ICA		SBA	
			No. of Colors	Time (minutes)	No. of Colors	Time (minutes)	No. of Colors	Time (minutes)
le450_15a	450	8.168	15	90	15	115	15	84
le450_5a	450	5.714	5	160	5	210	5	158
Myciel3	11	20	5	1s	4	2s	4	1s
Myciel4	23	71	5	1s	5	3s	5	1s
Myciel5	47	236	6	1s	6	1s	6	1s
Myciel6	95	744	7	8s	7	10s	6	8s
Myciel7	191	2.360	9	1	9	2	8	49s
Queen5_5	25	160	6	1s	5	3s	5	1s
Queen6_6	36	290	7	4s	7	10s	7	4s
Queen7_7	49	476	8	2	7	2	7	100s
Queen8_8	64	728	9	2	9	5	9	112s
Queen9_9	81	2112	10	2	10	3	10	100s
Queen15-15	225	10.360	17	18	16	32	16	15
Queen16_16	256	12.640	18	23	18	40	17	25
School11	385	19.095	14	85	14	100	14	84
School11_nsh	352	14.612	14	65	14	80	13	62
flat300_26_0	300	21.633	28	120	28	136	26	96

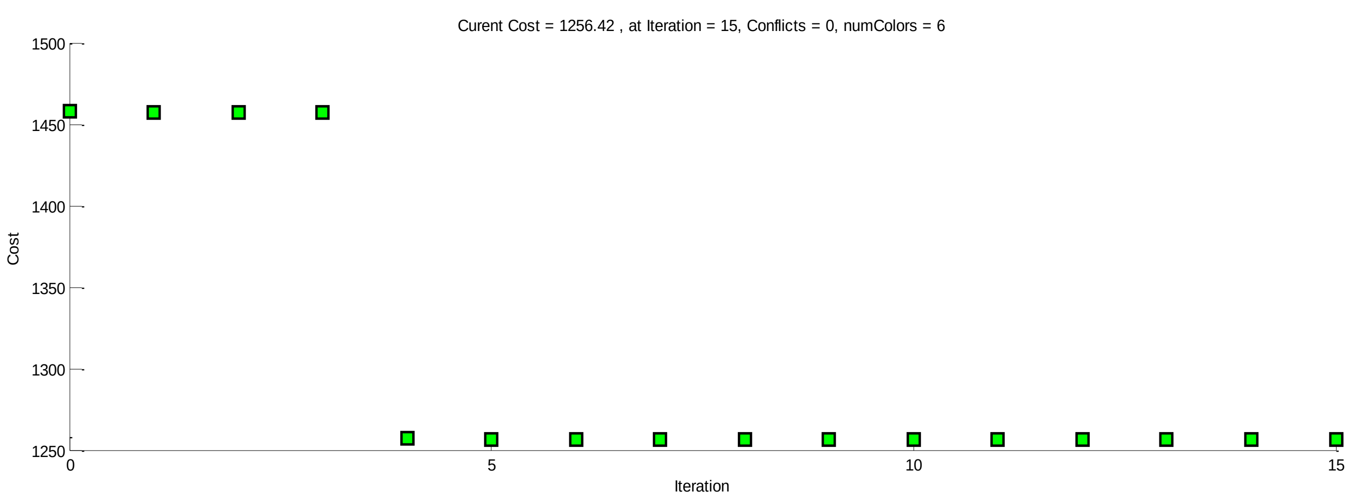

Fig. 2 - Algorithm convergence in the case of the Myciel6 graph benchmark.

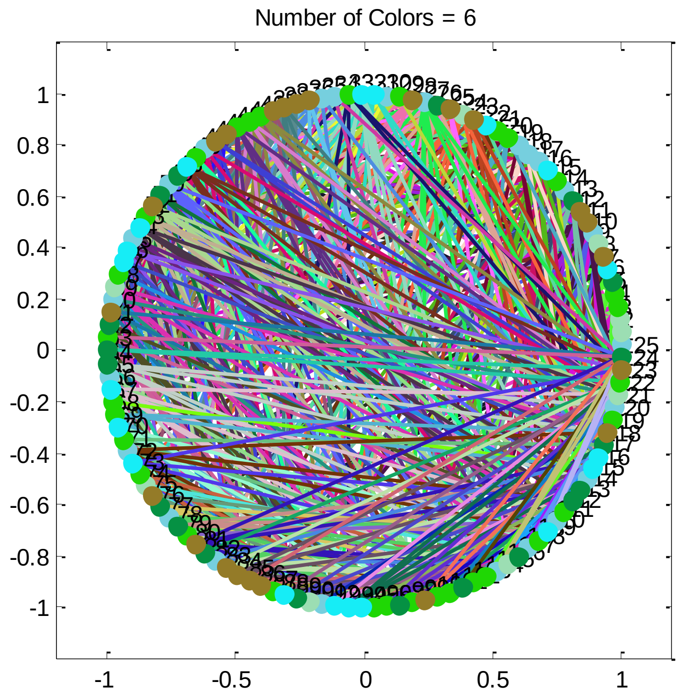

Fig. 3 - Algorithm movement in the case of the Myciel6 graph benchmark

CONCLUSION

In this article, we attempted to improve a new evolutionary algorithm, Social Based Algorithm (SBA) to graph coloring problem. For extension of SBA to discrete problem, we utilized a combination of discrete scaling and space mapping techniques over any discrete solution space. A distance is introduced as the least number of consecutive applications of the operator on the solution space. The description is a common concept provided a definite set of solutions and an operator on solution is given. Under this conception for distance, we improved the conventional SBA operators based on the basic idea of SBA. After redefining the parameters for SBA algorithm which is a combination of Genetic and Imperialist Competitive algorithms, we proposed a framework of SBA algorithm for any discrete problem. Experiment on a set of 17different benchmarks was evaluated and the computational results show that SBA is feasible and competitive with other well-known algorithms.

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