



# Modeling and Optimizing the Hardness of the Melted Zone in Submerged Arc Welding Process using Taguchi Method

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**Abstract** – Welding, as one of the most useful method for permanent joint of components, is of great importance in industry. Among the wide variety of welding processes, submerged arc welding, given its particular characteristics, is commonly used in industries. The distinguishing advantages of this method are high penetration and sedimentation rate as well as alloy development during welding by creating a cover of desired combinations of elements on the surface of work piece, which improve mechanical, corrosive, fricative and other properties. In this process, the proper selection of input parameters is necessary for high productivity and cost-effectiveness. One of the important characteristic of weld quality, which is influenced by welding parameters, is the hardness of the melted zone (HMZ). In this paper, experiments were conducted by Taguchi experimental design and Minitab 14 statistical software, and the interaction of input parameters was not taken into account. Current intensity, arc voltage, welding speed, nozzle distance from the work piece and thickness of magnesium oxide nanoparticles were considered as the input parameter and the HMZ was assumed as the response. After collecting data, the signal to noise ratio (S/N) was calculated to obtain optimal levels for all input parameters. Then, using analysis of variance (ANOVA), the significance level of (P) for each input parameter was determined and validated for the hardness of the melted area. The results show that current intensity, welding speed, arc voltage, nozzle distance from work piece and thickness of magnesium oxide nanoparticles had respectively the highest impact on the hardness of melted zone.

**Keywords:** Submerged Arc Welding, Hardness of Melted Zone, Taguchi Method, Analysis of Variance, Optimization

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## INTRODUCTION

One of the most common methods of industrial metal welding is submerged arc welding that is used for joining massive metal pipes in extensive lines of gas and oil transfer as well as huge metal pieces [1, 2].

The complexity of parameters in submerged arc welding on the one hand and their widespread use in manufacturing critical and expensive pieces on the other hand highlight the importance of precise control of input parameters. In recent years, with the increasing development of manufacturing industries, the quality of the products and efficiency of these industries has improved.

In welding processes, geometry and quality of welded joints is greatly dependent on the input parameters. In this regard, given the large number of parameters involved, the correct determination of input parameters is of paramount importance to achieve the

desired geometrical characteristics aimed at reducing costs of production and maintaining the quality of products. That is, obtaining a weld with the desired quality requires complete control and optimization of parameters involved in this process [3, 6].

There are several methods for achieving optimal parameters among which Taguchi design is one of the most efficient as it can reduce the number of tests. Taguchi design is a powerful method for reducing the production costs, improving quality and reducing the interval between incremental development process [7, 8].

Overall, it can be said that experimental design is one of the most basic scientific analysis methods and a powerful device for enhancing the quality of industrial products and achieving optimal conditions.

Commonly, a wide variety of statistical methods are used for research and development in industries, quality control and statistical studies with each having their merits and demerits. Among these methods, Taguchi

design has a distinctive position and enjoys an excellent reputation around the world [9, 11].

It is important to know how input welding parameters impact the hardness of the melted zone because it can be used for automatic and semi-automatic control of submerged arc welding process.

Thus, many studies have been carried out to understand the relationship between the input parameters of submerged arc welding and HMZ. Yang [12] reported that the hardness of the melted zone increased with decreasing inlet temperature. Hall [13] reported that welding speed could influence HMZ in submerged arc welding process. Kolhe and Datta [14] investigated the effect of welding parameters on the HMZ concluding that the hardness of the weld was a variable of input thermal variation. Hashemi and the Mohammedan [15] investigated the effect of welding parameters on the hardness of the fusion zone (FZ) of API X65 steel concluding that FZ was affected by the variation in the melting zone.

In this paper, the effect of welding input parameters on the HMZ in submerged arc welding process was studied. Therefore, the arc voltage (V), the intensity of welding current (I), nozzle distance from the work piece (N), welding speed (S) and the thickness of magnesium oxide nanoparticles (F) were determined as input parameters and HMZ was considered as the response

variable. The results show that Taguchi method is able to predict HMZ with lower error.

## MATERIAL AND METHODS

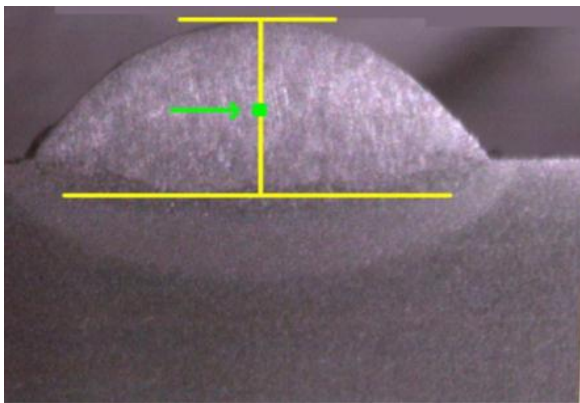
### Test method

To weld pieces, the surface of work piece was covered with magnesium oxide nanoparticles. Then, the welding process was carried out using PARS FEED 1202G semi-automatic machine with direct current reverse polarity (DCRP) on the surface through bead on plate welding method. Work pieces were made of St37 steel with 15 mm × 50 mm × 150 mm dimensions and (DIN EN 756) S1 welding wire with a diameter of 3.2 mm. The chemical composition of welding wire is shown in Table 1.

After welding, the pieces were cut perpendicular to the weld line in a distance of 70 mm from the edge using Wire Cut machine. The sheared cross section was first abraded by 240, 320, 400, and 600 abrasive tools and then polished by 6 micrometer alumina. Then, the cross-section of pieces was etched by 2% solution of Nital for 1 min. Having prepared the pieces, the hardness of the melted zone was measured by Instron Wolpert hardness gauge using Vickers method with a force of 20 kN. For this purpose, a point was used as the average point in the melted zone (MZ), as shown in Figure 1.

**Table 1.** Chemical composition of the welding wire

Type of welding wire		Weight percent				
Brand	DIN/EN	Carbon	Silicon	Manganese	Molybdenum	Chromium
50-11	S1	0.04-0.08	0.5-0.8	0.9-1.3	-	-



**Figure1.** Average point in HMZ calculation [16].

### Experiment Design based on the Taguchi method

Experiments are used extensively in various disciplines such as engineering sciences. The aim of an experiment is to investigate the effect of input parameter and to offer a model that obviates the need for the replication of the experiment under the same circumstances, and thus reduce the costs and time and improve the efficiency.

Accordingly, the aim of this study is to optimize and model input parameters of the submerged arc welding process using Taguchi experiment design. Taguchi is an experiment analysis method based on which one can predict the effect of factors and optimization levels of empirical studies with only a certain number of tests.

This method is widely used in automobile industry, electronics, and other processes [17]. Therefore, based on the orthogonal arrays of L25, and for five input parameters of the intensity of welding current, arc voltage, nozzle distance from the work piece, the welding speed and the thickness of nanoparticles of magnesium oxide (MgO) in five levels, the output parameters of HMZ St37 steel were evaluated by conducting 25 experiments.

The values of the input parameters at different levels are shown in Table 2, design matrix and Table 3. Also, considering the values of signal to noise ratio (S /N) and analysis of variance (ANOVA), the optimum levels of the input parameters were determined. All analyses were performed by Minitab 14 Statistical Software and the interaction of input parameters were ignored in this study.

**Table 2.** Welding parameters and their range

Unit	Coding					Symbol	Input Variable
	5	4	3	2	1		
Ampere	700	650	600	550	500	I	Current Intensity
Volt	32	30	28	26	24	V	Arc Voltage
Mm	40	37.5	35	32.5	30	N	Nozzle Distance From The Work Piece
Mm/min	500	450	400	350	300	S	Welding Speed
Mm	1	0.75	0.5	0.25	0	F	Thickness Of Nanomaterial

**Table3.** Design Matrix

Level	V	I	N	S	F	Level	V	I	N	S	F
1	1	1	1	1	1	14	3	4	1	3	5
2	1	2	2	2	2	15	3	5	2	4	1
3	1	3	3	3	3	16	4	1	4	2	5
4	1	4	4	4	4	17	4	2	5	3	1
5	1	5	5	5	5	18	4	3	1	4	2
6	2	1	2	3	4	19	4	4	2	5	3
7	2	2	3	4	5	20	4	5	3	1	4
8	2	3	4	5	1	21	5	1	5	4	3
9	2	4	5	1	2	22	5	2	1	5	4
10	2	5	1	2	3	23	5	3	2	1	5
11	3	1	3	5	2	24	5	4	3	2	1
12	3	2	4	1	3	25	5	5	4	3	2
13	3	3	5	2	4						

**RESULTS AND DISCUSSION**

**Properties of melted zone**

Heat transfer from the melting pool to the base metal is one of the most important steps in cooling and solidification processes. In this stage, with the transfer of heat to the base metal in the vicinity of MZ through welding process, several metallurgical processes occur that affect the mechanical properties of weld joint. That is, they reduce its tensile and impact resistance, leading to an increased hardness or crack formation.

In fact, MZ is obtained from the melting of the base metal and filler metal with a combination that is often different from the base metal [18, 19].

St37 steels tend to become hardened in this zone. The hardening primarily depends on the chemical composition of the steel and the applied procedure. These steels have a number of alloying elements. The major concern about the hardening of this zone is formation of cracks which can significantly reduce the coupling resistance. The measurement of hardening can provide information about metallurgical changes caused by welding [20, 21].

**S/N ratio**

To obtain the sensitivity of parameters and optimal conditions in welding process, Taguchi method was used as it offered a way of improving the quality of products through reducing change effects without eliminating their causes [22, 24].

In this method, the process parameters are divided into controllable and uncontrollable parameters. For

example, in submerged arc welding, current intensity, arc voltage, welding speed, nozzle distance from the work piece and similar factors fall in controllable group. Uncontrolled parameters are all factors that can change, but since they are difficult to control or there is little information about them, they are kept constant in certain cases. For example, in the submerged arc welding, machine vibrations, variation of parameters given to the machine during welding, temperature, humidity and other environmental factors are among uncontrolled factors. Taguchi design draws on S/N ratio analysis for the analysis of experiment results, which indicates the sensitivity of the target feature to the input factors in a controlled process.

Optimal conditions are identified through the effect of each input factor on the output feature [25]. The aim of S/N ratio analysis is to determine the best combinations of factors at various levels to achieve the optimal response. It is defined in three different states as follows:

"Nominal value – better" (NB) "lower value - better (LB)" and "higher value - the better (HB)"

In this paper, LB value was used to achieve desirable results for the hardness of the melted zone.

(Eq. 1) the lower, the better

$$SN = -10 \log \left( \frac{1}{n} \sum_{i=1}^n Y_i^2 \right) \tag{1}$$

In this equation, Yi is the response of output value and n is the number of experiments.

Using the experimental values of output parameters, the values of S/N ratio for the hardness of melted zone parameter was calculated (Table 4). In Table 5, the ratio

of efficient to inefficient function (S/N) was calculated for each control factor level. Because the optimal value for the hardness of the melted zone was the lower value (the lower, the better), by comparing the value of this ratio for the levels of each factor, the lowest value represented the optimal level of each factor. As indicated by S/N ratio in

the Table, Delta and Rank reflects the importance of a parameter in the process. High Delta value for a parameter indicates the importance of that parameter in the process. However, high Rank value reduces the importance and the impact of the parameter on the desired output.

**Table 4.** Experimental values of the output parameters and the values of S/N ratio

Level	Harness Of Melted Zone	S/N Ratio	Level	Harness Of Melted Zone	S/N Ratio
1	172	44/7106-	14	161	44/1365-
2	175	44/8608-	15	155	43/8066-
3	183	45/2490-	16	143	43/1067-
4	191	45/6207-	17	127	42/0761-
5	183	45/2490-	18	155	43/8066-
6	175	44/8608-	19	161	44/1365-
7	168	44/5062-	20	148	43/4052-
8	175	44/8608-	21	148	43/4052-
9	161	44/1365-	22	137	42/7344-
10	168	44/5062-	23	143	43/1067-
11	175	44/8608-	24	148	43/4052-
12	158	43/9731-	25	137	42/7344-
13	148	43/4052-			

In the current intensity factor, the first level has the highest S/N ratio. Thus, 500 amps current is the optimum level for this factor. As shown in the Table, based on the Rank of parameters specified in the table of parameters involved in the hardness of melted zone, current intensity, arc voltage, nozzle distance from work piece, speed and thickness of the nanoparticles are respectively significant.

**Table5.** Moderate response of S/N ratio for the hardness of the melted zone

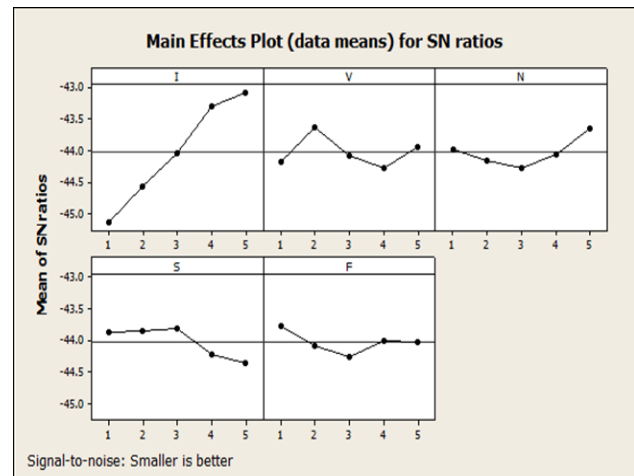
F	S	N	V	I	level
-43/77	-43/87	-43/98	-44/19	-45/14	1
-44/08	-43/86	-44/15	-43/63	-44/57	2
-44/25	-43/81	-44/29	-44/09	-44/04	3
-44/01	-44/23	-44/06	-44/29	-43/31	4
-44/02	-44/37	-43/65	-43/94	-43/08	5
0/48	0/56	0/63	0/66	2/06	Delta
5	4	3	2	1	Rank

In addition, the impact of input parameters on S/N ratio is shown in Figure 2. The higher point represents greater value of S/N ratio. The current in the fifth level, the arc voltage in the second level, nozzle distance from the work piece in the fifth level, speed in the third level and the thickness of the nanoparticles in the first level had the highest S/N ratio. These results are a graphical representation of the results of the above Table.

As shown in the diagram, increased current intensity strengthens the harness of melted zone and the effect of other parameters on this mechanical property follows a regular trend. That is, with the increased distance of nozzle from work piece, this zone can be rising and

declining. Moreover, it can have a declining-rising state for two variables of thickness of nanoparticles and speed and a rising-declining-rising state for arc voltage.

According to this figure, the optimal state is achieved at I5V1N5S3F1 stage.



**Figure 2.** Effect of input parameters on S/N ratio

**Analysis of Variance**

In order to validate and select the most efficient models, the statistical tests are used. Table 6 shows the analysis of variance for the hardness of the melted zone. Based on statistical analysis of the data, this method can show which parameters have the most important and the greatest effect on output parameters. In ANOVA tables, the p value is important. That is, the low value of p indicates that the input parameter has greater impact and significance on output parameter. In this article, when p

value is lower than 0.05, the input and output parameters will have their greatest effect. We chose 95% confidence

level for this matter [26].

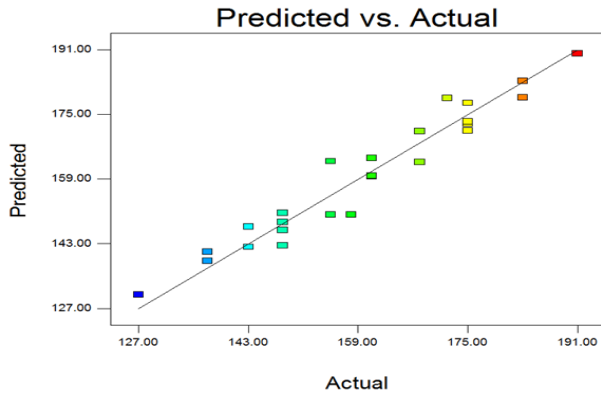
**Table 6.** Analysis of variance parameters for the hardness of the melted zone

Source	DF	Seq SS	Adj SS	Adj MS	F	P
I	4	4990/8	4990/8	1247/70	23/59	0/005
V	4	394	394	98/50	1/86	0/281
N	4	342/8	342/8	85/70	1/62	0/326
S	4	436/4	436/4	109/10	2/06	0/250
F	4	172/4	172/4	43/10	0/81	0/576
Error	4	211/6	211/6	52/90		
Total	24	5365/75				

DF = Degree of Freedom, Seq SS = Sequential Sum of Squared; Adj SS = Adjusted Mean Squared; Adj MS = Adjusted Mean Squared; F = Fisher's F ratio; P = Probability of Significance; S = 273/7 R-Sq = 96.8% R-Sq (adj) = 80.6%

According to Table 6 and p value, the parameters influencing HMZ are respectively current intensity, arc voltage, nozzle distance from work piece and thickness of nanoparticles.

Another important criterion for evaluating the accuracy and quality of the fitted modes is the correlation coefficient. This coefficient shows the degree of dependency between input and output parameters of a system. As the value of the coefficient leans toward 1, the dependence of input and output parameters is increased [27]. Figure 3 shows the correlation of data and predictions of Taguchi method. As can be seen, this value is 80.6%, which has been extracted by Design Expert 7 software.



**Figure 3.** Diagram of predicted values versus actual values for the hardness of the melted zone

**The results of confirmatory analysis for the hardness of the melted zone**

Confirmatory test by considering the specific combination of factors and levels, which were already identified through a series of calculations as the optimal combination, guarantee the accuracy or inaccuracy of results and decision makings. If the mean value of confirmatory test results is in the confidence level, they are confirmed; otherwise, the results are rejected,

meaning that the significant parameters are not selected, factors are not positioned at a proper level or calculations and experiments have high error rate. In Table 7, five experiments outside the design are given along with a confirmatory analysis of the results.

Table 8 shows the confirmatory results of the test for the hardness of the melted zone. As can be seen, the errors obtained for the hardness of the melted zone are within an acceptable range with the results suggesting that Taguchi design experiment is able to predict external values of the design for the hardness of the melted zone.

**Table 7.** Factors input for confirmatory testing.

level	I	V	N	S	F
1	1	4	5	3	2
2	2	4	4	1	3
3	5	2	3	4	1
4	4	1	5	2	3
5	3	2	1	5	4

**Table 8.** The results of confirmatory test for the hardness of the melted zone

Error	Prediction	Experiment	No
1/88	176/6	180	1
7/60	175/4	163	2
4/38	139/6	146	3
2/97	143/6	148	4
7/72	157/8	181	5
4/91	<b>Average</b>		

**Optimization**

In this article, the optimal value for the hardness of the melted zone can be calculated by utility function. For this purpose, using Minitab 14 software, the overall utility function is introduced based on utility functions of means and variances of five input parameters at five levels. The results are presented in Table 9. Also, the optimal solution for five effective parameters is given in Table 9. In this



method, the utility of each purpose is determined and then using a method like geometric mean, the total utility is calculated. If the utility function  $y_i(x)$  is a monotonically increasing, the utility value is calculated according to Eq. 2 [29]:

$$d_i = \begin{cases} 0 & \hat{y}_i(x) \leq Y_i^{min} \\ \left[ \frac{\hat{y}_i(x) - Y_i^{min}}{Y_i^{max} - Y_i^{min}} \right]^t & Y_i^{min} \leq \hat{y}_i(x) \leq Y_i^{max} \\ 1 & \hat{y}_i(x) \geq Y_i^{max} \end{cases} \quad (2)$$

Where

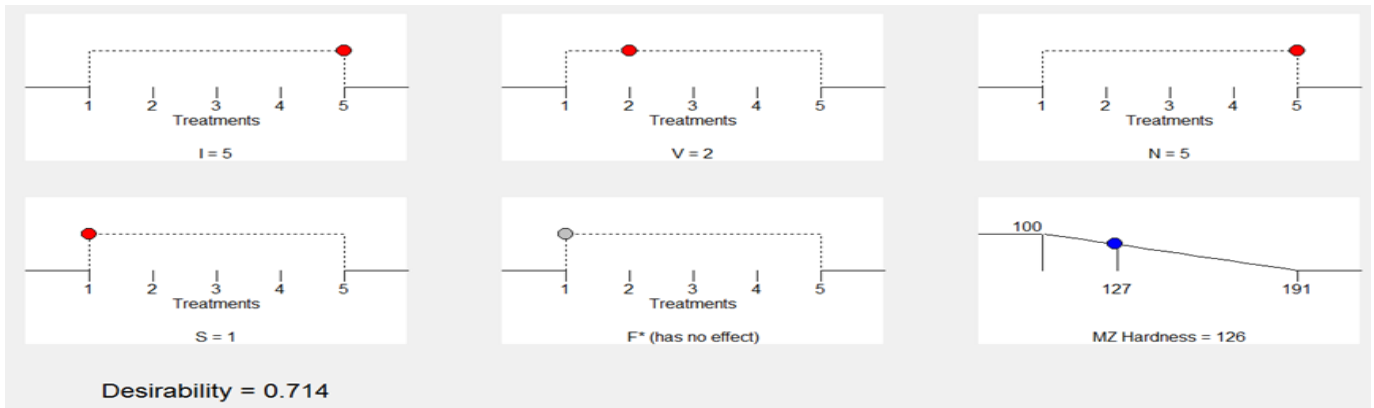
$\hat{y}_i(x)$  is an estimate of  $y_i(x)$  and  $Y_i^{min}$  is the minimum acceptable value of  $y_i(x)$  from the view of decision-makers,  $Y_i^{max}$  is the value of  $y_i(x)$  with the maximum utility for the decision maker and after which the utility is constant, and  $t$  is the parameter that determines the form of the utility function.

Although utility function is an efficient method, it has its own problems. One problem is the difficulty of drawing indifference curves and determining utility. Determining utility relative to the view of the decision maker is critical. On the other hand, the above techniques are only useful when the utility of dependent variables is monotonically rising or declining. In other words, the greater (or lower) is the response variable, the higher is desirability. However, it is always possible that the utility function is quadratic. In this case, the most appropriate value of a response may be in the middle of variation range.

The optimal conditions of factors in the range of selected levels include a current intensity of 700 amps, an arc voltage of 24 V, a 30 mm distance of nozzle from work piece, a welding speed of 400 mm/min and a 0 mm thickness of nanoparticles. Under these circumstances, the minimum hardness for the hardness of melted zone is 126 Vickers with a desirability of 0.174, as shown in Figure 4.

**Table 9.** The optimal values of the input parameters for the hardness of the melted zone

I	V	N	S	F	Min hardness of the melted zone
5	2	5	1	1	126 VHN



**Figure 4.** The utility function for hardness of the melted zone

### CONCLUSION

This study sought to find the optimal values for improving the welding quality and the desired mechanical properties. At the end, after taking all steps, the effect of each input parameter on the hardness of the melted zone is as follows:

1. According to S/N mean diagrams and variance analysis, the optimal state for the hardness of the melted zone (the higher, the better) was achieved in I5V2N5S1F1.
2. According to the results of table and confidence level of 95%, the input parameters of current intensity, speed, arc voltage, nozzle distance from the work piece

and the thickness of nanoparticles on the hardness of the melted zone had the greatest impact.

3. According to the results of variance analysis, the correlation coefficient of 80.6% and the utility value of 0.714 were achieved.

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